one would immediately ask whether this formulation may be used for similar large deflection problems such as buckling. The answer is that although for a certain class of problems this may indeed be advantageous, the general applicability is limited by the fact that if boundary conditions are imposed on the radial displacement

$$v(\ell) = \zeta(\ell) - \frac{1}{2}\int_0^\ell \left[\frac{\partial\xi}{\partial y}\right]^2 \mathrm{d}y \tag{3}$$

then the boundary condition becomes nonlinear although the differential equations remain linear. This point also underscores the general necessity of considering simultaneously the flexural and axial degrees of freedom in large deflection problems. It should also be pointed out that the Vigneron formulation trades the advantage of kinematic orthogonality ($\dot{\xi}$ and $\dot{v}$ are orthogonal velocity componets, $\dot{\xi}$ and $\dot{\zeta}$ are not) for structural simplicity. Conceivably there may be problems for which such trades are not worthwhile.

This commentator is in agreement with Professor Likins that the "partial linearization" method will remain one of the most useful methods. Unfortunately the radial beam example given in Ref. 1 represents a somewhat incorrect application of the method. Dr. Vigneron's formulation is certainly a viable alternative approach which deserves further exploration.

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Reply to Bertrand T. Fang

F.R. Vigneron*
Communications Research Centre, D.O.C., Ottawa, Canada

DR. FANG correctly points out that the method outlined in Ref. 2 does not depend on the suppression of the extensional displacement, ζ. The suppression is a convenient approximation, which may be introduced to shorten the derivation somewhat.

Dr. Fang further notes that the boundary condition on $v(\ell)$ is nonlinear (Eq. (3) of his Comment). This does not necessarily preclude the possibility of use of the formulation for buckling problems. As an example, one may consider the case where Ω and $\partial(\ )/\partial t$ are equal to zero, bending is in one plane only ($\eta=0$), and a constant axial load, P, is applied at $y=\ell$. The work done by the load (negative potential) is the proportional to $-Pv(\ell)$, or

$$-P\{\zeta(\ell) - \tfrac{1}{2}\int_0^\ell \left(\frac{\partial\xi}{\partial y}\right)^2 \mathrm{d}y - \zeta(0)\}$$

Application of the principle of virtual work, with the above and the correspondingly simplified potential of Eq. (6), Ref. 2, leads to

$$EA\,\zeta_y = 0$$

$$EI\,\xi_{yyyy} - P\,\xi_{yy} = 0,$$

together with appropriate boundary conditions. The latter equation may be recognized as that associated with the study of buckling of beam columns. More general extensions of the formulation along these lines are given in Ref. 2.

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*Research Scientist. Member AIAA.

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Comment on "On the Issue of Resonance in an Unsteady Supersonic Cascade"

Joseph M. Verdon*
United Technologies Research Center, East Hartford, Conn.

VERDON and McCune[1] have reported that the iterative procedures used in their linear unsteady supersonic cascade analysis failed to converge for certain combinations of the cascade parameters. The range in which divergence occurred was given as

$$|\sigma + kMx_A| \leq k(x_A^2 - \mu^2 y_A^2)^{1/2} \tag{1}$$

where $k=\omega M\mu^{-2}$, $\mu^2 = M^2 - 1$, M is the freestream Mach number, x_A and y_A are the cascade stagger and normal gap distances, respectively, and ω and σ are the frequency and interblade phase angle of the blade motion. (Note that only an interblade phase angle variation of 2π is of physical interest and hence σ can be restricted to the range $-\pi<\sigma\leq\pi$.) Since the publication of Ref. 1, numerical procedures have been developed by the present author which provide results within the foregoing range, but not at its end points

$$\sigma + kMx_A = \pm k(x_A^2 - \mu^2 y_A^2)^{1/2} \tag{2}$$

At these points the numerical approach used for evaluating the sum of an infinite series kernel function, given by Eq. (24) of Ref. 1, fails. On this basis it was suspected that this infinite series might be divergent for such parametric combinations, indicating resonant operation; however, this conjecture could not be proved. Thus, the proof appearing in Ref. 2 on the divergence of the kernel function for parameter values satisfying Eq. (2) is a most welcome contribution. This work is particularly useful since it generalizes the earlier work of Samoilovich,[3] which was only recently brought to this author's attention by Kurosaka. Further, Samoilovich's result appears to have been achieved by a formal mathematical demonstration rather than by a rigorous proof.

Resonance appears to be the logical consequence of linear theories. The conditions given by Eq. (2), or more generally by Eq. (10) of Ref. 2, are formally identical to those obtained for a subsonic cascade. In addition, resonance has the same physical interpretations for both subsonic and supersonic flows, including the result that the blades cannot support un-

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*Senior Research Engineer, Aeroelastics Group. Member AIAA.

steady loading. However, since resonance depends on the propagation of unsteady disturbances into the far field and it is not clear that a linear supersonic formulation can accurately simulate far-field behavior, the linear model may not be adequate for actual cascades or fans operating near predicted resonance conditions. The present author disagrees with the contention expressed in Ref. 2 that a linear analysis using a passage approach can be used to circumvent this problem. In a passage approach the blade-to-blade periodicity requirement must be supplemented by prescribing information on an upstream boundary of a given blade passage which must be satisfied by the velocity potential. Such information depends on the unsteady disturbances generated by the infinite array of blades below the reference passage. If the potential distribution on the prescribed upstream boundary is determined on the basis of linear equations, as in Ref. 4, and the linear formulation possesses a unique solution, then resonant points should be predicted in the final solution.

It appears that the only way to resolve the resonance problem is to introduce nonlinear effects into the governing equations in an attempt to model far-field behavior properly. This has been done for the case of steady supersonic flow past a thin, isolated airfoil. Although Ackeret's classical linearized solution is a proper first approximation in the near field, it fails at great distances from the airfoil. Ackeret's solution predicts disturbances propagating undiminished along the freestream Mach lines to infinity, whereas in reality the Mach lines are not straight and parallel. In this situation the nonlinear terms are small compared with the linear ones; however, their cumulative contribution gives rise to a first-order effect as the distance from the airfoil increases. A uniformly valid first-order approximation has been achieved by adding a nonlinear term, called the "pseudo-transonic" term, to the differential equation governing the velocity potential.[5] A solution to the resulting boundary value problem, obtained by the method of strained coordinates, gives velocity components that are the same function of distance along revised Mach lines as predicted by Ackeret's solution along the freestream Mach lines.

Since the flow adjacent to a given blade of a supersonic cascade with subsonic axial flow is influenced by the far-field disturbances from the blades below, it may prove necessary to incorporate similar considerations into an unsteady supersonic cascade analysis. However, up to the present time the linear formulation has been quite successful at predicting the measured flutter behavior of supersonic test fans.[6,7] Of course this success could be due to the inherent stability of these fans when operating near predicted resonance and further experimental work is necessary for clarification. The introduction of nonlinear terms into an unsteady analysis would lead to complications over and above those encountered in the foregoing steady example. When nonlinear terms are included, time dependence can not be eliminated from the resulting boundary value problem by simply removing the factor $e^{i\omega t}$ from each term in the governing equations.[1] In addition to the fundamental excitation at the blade motion frequency, higher harmonic excitations must also be considered. If time dependence cannot be avoided in a nonlinear formulation, one of the major features, i.e., computational efficiency, which renders a flutter analysis useful to the designer, will be lost. Obviously the resonance issue raised by Kurosaka[2] requires some further work before a satisfactory resolution will be achieved.

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Comment on "Bolotin's Method Applied to the Buckling and Lateral Vibration of Stressed Plates"

Wilton W. King*

Georgia Institute of Technology, Atlanta, Ga.

DICKINSON[1] has applied Bolotin's edge effect method to vibration and buckling of rectangular orthotropic plates subjected to uniform in-plane loads. The successful estimation of buckling loads was particularly interesting to the writer, since he had once attempted the same analysis of a similar problem and found the method to be inapplicable. The purpose of this Comment is to point out that the edge effect method is not universally applicable when one or both of the in-plane loads is compressive.

It is useful to reproduce, from Dickinson's paper, the frequency equation

$$\omega^2 = (1/\rho)\,[D_x (k_x/a)^4 + 2H(k_x k_y/ab)^2 + D_y (k_y/b)^4 + N_x (k_x/a)^2 + N_y (k_y/b)^2]$$

and the equations for the parameters associated with that part of the transverse displacement that decays away from the edges,

$$\gamma_x^2 = k_x^2 + 2(H/D_x)\,(k_y a/b)^2 + N_x a^2/D_x$$

$$\gamma_y^2 = k_y^2 + 2(H/D_y)\,(k_x b/a)^2 + N_y b^2/D_y$$

For Bolotin's method to work, γ_x^2 and γ_y^2 must be positive, which is clearly the case if neither N_x nor N_y is negative (in-plane compression). If either N_x or N_y is negative it is not clear, in advance of determining the wave numbers k_x and k_y, whether or not the edge effect is, in Bolotin's[2] words, "degenerate."

If we restrict our interest to an analysis of buckling ($\omega^2 = 0$), the frequency equation may be solved for the buckling loads,

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*Associate Professor, School of Engineering Science and Mechanics.